Modes of persuasion toward unanimous consent (Online appendix)

Arjada Bardhi* Yingni Guo†

B.1 Proofs for sections 5 and 6

Proof for lemma 5.1.

(i) General persuasion: Let $(\pi(\hat{d}^a|\theta))_\theta$ be an optimal policy with n voters. Suppose a voter R_i is removed from the group, so there are only $(n-1)$ voters left. The distribution of states for the remaining voters is given by $\tilde{f} : \{H, L\}^{n-1} \to [0,1]$ such that $\tilde{f}(\theta_{-i}) = f(\theta_{-i}, H) + f(\theta_{-i}, L)$. We construct $\tilde{\pi}(\hat{d}^a|\cdot) : \{H, L\}^{n-1} \to [0,1]$ such that for any $\theta \in \{H, L\}^{n-1}$ and $\theta', \theta'' \in \{H, L\}^n$ with $\theta'_j = \theta''_j = \theta_j$ for $j \neq i$ and $\theta'_i = H$, $\theta''_i = L$:

$$\tilde{f}(\theta)\tilde{\pi}(\hat{d}^a|\theta) = f(\theta')\pi(\hat{d}^a|\theta') + f(\theta'')\pi(\hat{d}^a|\theta'').$$

Notice that the IC^a constraints of the remaining voters are satisfied, as

$$\frac{\sum_{\theta\in\{H,L\}^{n-1}:\theta_j=H} \tilde{f}(\theta)\tilde{\pi}(\hat{d}^a|\theta)}{\sum_{\theta\in\{H,L\}^{n-1}:\theta_j=L} \tilde{f}(\theta)\tilde{\pi}(\hat{d}^a|\theta)} = \frac{\sum_{\theta\in\{H,L\}^{n}:\theta_j=H} f(\theta)\pi(\hat{d}^a|\theta)}{\sum_{\theta\in\{H,L\}^{n}:\theta_j=L} f(\theta)\pi(\hat{d}^a|\theta)} \geq \ell_j.$$

By the same reasoning, the sender attains the same payoff under $\tilde{\pi}$ as under π. Hence, he is weakly better off with $(n-1)$ voters.

(ii) Individual persuasion: Consider a group of n voters, characterized by a threshold profile $\{\ell_1, ..., \ell_n\}$ with $\ell_i > \ell_{i+1}$ for any $i \leq n-1$. By proposition 4.2 there exists a monotone optimal policy. We let $(\pi_i(L))_{i=1}^n$ denote this policy. Suppose that the sender does not need some voter's approval, so we are left with $n-1$ voters. We relabel the remaining voters monotonically, so that R_1 is the strictest and R_{n-1} is the most lenient voter. In the first step, we assign the original $\pi_i(L)$ to the ith strictest voter among the remaining $n-1$ voters. Therefore R_n's original policy will be removed from the group.

*Department of Economics, Northwestern University, abardhi@u.northwestern.edu.
†Department of Economics, Northwestern University, yingni.guo@northwestern.edu.

Note that the remaining $n-1$ voters might not be willing to obey the approval recommendation given the policy $\{\pi_1(L), ..., \pi_{n-1}(L)\}$ if we remove R_n's original policy $\pi_n(L)$ from the group. This is because voter $R_j \neq R_n$ might be willing to approve only when she conditions the approvals by the other voters including R_n. We need to construct a new policy for the smaller group $\{R_1, ..., R_{n-1}\}$. The main step is to show that the informativeness of $\pi_n(L)$ can be loaded into the policies of the remaining voters so as to satisfy the remaining voters' IC^a constraints when $\pi_n(L)$ is removed from the policy profile.

Our first observation is that if $\pi_n(L) = 1$, then the policy $\{\pi_1(L), ..., \pi_{n-1}(L)\}$ is incentive compatible since each voter in the smaller group $\{R_1, ..., R_{n-1}\}$ is assigned a (weakly) more informative policy than before. Moreover, removing R_n's policy has no impact on the other voters' incentive constraints since she always approves. We focus on the case in which $\pi_n(L) < 1$. We want to show that we can adjust the policy $\{\pi_1(L), ..., \pi_n(L)\}$ in an incentive compatible way so as to increase $\pi_n(L)$ until it reaches one. At that point, removing $\pi_n(L)$ from the policy profile will not affect the IC^a constraints of $\{R_1, ..., R_{n-1}\}$.

Since R_n will eventually be removed, this is as if R_n's IC^a constraint is always slack. In proposition 4.3, we show that among those voters who have interior $\pi_i(L)$, only the strictest voter might have a slacking IC^a constraint. In our current setting, R_n's policy corresponds to the most lenient voter's policy and R_n's IC^a constraint is slack. Therefore, we follow the reasoning in the proof of proposition 4.3 to show that we can always increase $\pi_n(L)$ and decrease another voter's policy in an incentive-compatible way without decreasing the sender's payoff. In particular, if $\pi_{n-1}(L)$ is also interior, we can replace $(\pi_{n-1}(L), \pi_n(L))$ with $(\pi_{n-1}(L) - \varepsilon_2, \pi_n(L) + \varepsilon_1)$ so that the sender is better off and the IC^a constraints of $\{R_1, ..., R_{n-1}\}$ are satisfied. Since R_n's IC^a constraint is constantly slack, we can make this adjustment on the pair $(\pi_{n-1}(L), \pi_n(L))$ until either $\pi_n(L)$ reaches one or $\pi_{n-1}(L)$ reaches zero. If $\pi_n(L)$ reaches one before $\pi_{n-1}(L)$ drops to zero, we are done with the construction, because removing R_n from the group now will not affect the remaining voters' IC^a constraints. If $\pi_{n-1}(L)$ drops to zero before $\pi_n(L)$ reaches one, we can then adjust the pair $(\pi_{n-2}(L), \pi_n(L))$ by increasing $\pi_n(L)$ and decreasing $\pi_{n-2}(L)$ in a similar manner.

We keep making this adjustment until $\pi_n(L)$ reaches one: this is always possible if $\pi_i(L) > 0$ for some $i \in \{1, ..., n-1\}$. In other words, it cannot be that the policies of all other voters become fully revealing before $\pi_n(L) = 1$, as it takes an infinite amount of information from R_n's policy to push all other voters' policies to fully revealing ones. When $\pi_n(L)$ reaches one, removing R_n will not affect the remaining voters' IC^a constraints. If $\pi_j(L) = 0$ for $j \in \{1, ..., n-1\}$ in the original policy to begin with, removing R_n's policy will not affect the remaining voters' IC^a constraints since they all learn their states fully. This completes the construction. $\square$

Proof for lemma 5.2. From the proof of proposition 4.4, we know that given a perfectly correlated distribution f', for sufficiently correlated states i.e. for some f within $\varepsilon > 0$ of f', in any optimal individual policy IC^a-1 is the only binding IC^a-i constraint. The sender achieves the same payoff as if he needed only R_1's approval.

We next argue that this is also the optimal policy under general persuasion. From lemma 5.1, we know that the sender's payoff weakly decreases in the number of approvals he needs for a fixed threshold profile. Therefore, the sender's payoff cannot exceed the payoff from persuading only R_1. On the other hand, the sender can use the individual policy specified in the previous paragraph to achieve this payoff when the sender is allowed to use any general policy. Therefore, the probability of approval is equal for each θ across the two modes. $\square$

Proof for propositions 6.1. Suppose that $\hat{d}$ is observed publicly; we want to show that each R_i continues to comply with $\hat{d}_i$. Consider first $\hat{d} \in \{\hat{d}^a, \{\hat{d}^{r,i}\}_i\}$. By incentive compatibility of the optimal policy π, it follows immediately that each R_i follows her own recommendation, i.e. she complies with $\hat{d}_i$. Suppose now that $\hat{d} \notin \{\hat{d}^a, \{\hat{d}^{r,i}\}_i\}$. If all other voters R_{-i} follow the recommendation, R_i's vote is not pivotal. For any such $\hat{d}$ with two or more rejections, the project is not approved. Hence, R_i is indifferent between $\hat{d}_i$ and the other available action, as both yield an ex-post payoff of zero. $\square$

Proof for proposition 6.2. Consider first the case of individual persuasion. Take $(\pi_i)_i$ to be an optimal policy under simultaneous voting. Our first claim is that any policy that is incentive-compatible under simultaneous voting is incentive-compatible also under sequential voting, for a fixed voting order $\{1, ..., n\}$. Conversely, any policy that is incentive-compatible under sequential voting is incentive-compatible under simultaneous voting as well. It is sufficient to show that IC$^a_{sim}$-i, IC^a_{seq}-i, IC$^r_{sim}$-i, and IC$^r_{seq}$-i, are pairwise equivalent:

$$\left(\sum_{\Theta_i^H} f(\theta) \prod_{j \neq i} \pi_j(\theta_j) \pi_i(H)\right) \geq \ell_i \left(\sum_{\Theta_i^L} f(\theta) \prod_{j \neq i} \pi_j(\theta_j) \pi_i(L)\right), \qquad (IC^a_{sim}\text{-}i)$$

$$\left(\sum_{\Theta_i^H} f(\theta)\prod_{j=1}^{i-1}\pi_j(\theta_j)\pi_i(H)\prod_{j=i+1}^{n}\pi_j(\theta_j)\right) \geq$$
$$\ell_i\left(\sum_{\Theta_i^L} f(\theta)\prod_{j=1}^{k-1}\pi_j(\theta_j)\pi_i(L)\prod_{j=k+1}^{n}\pi_j(\theta_j)\right), \quad (IC^a_{seq}\text{-}i)$$

$$\left(\sum_{\Theta_i^H} f(\theta)\prod_{j\neq i}\pi_j(\theta_j)(1-\pi_i(H))\right) \quad \geq \quad \ell_i\left(\sum_{\Theta_i^L} f(\theta)\prod_{j\neq i}\pi_j(\theta_j)(1-\pi_i(L))\right), \quad (IC^r_{sim}\text{-}i)$$

$$\left(\sum_{\Theta_i^H} f(\theta)\prod_{j=1}^{i-1}\pi_j(\theta_j)(1-\pi_i(H))\prod_{j=i+1}^{n}\pi_j(\theta_j)\right) \geq$$
$$\ell_i\left(\sum_{\Theta_i^L} f(\theta)\prod_{j=1}^{k-1}\pi_j(\theta_j)(1-\pi_i(L))\prod_{j=k+1}^{n}\pi_j(\theta_j)\right). \quad (IC^r_{seq}\text{-}i)$$

The constraints are pairwise equivalent for simultaneous and sequential voting. Therefore, the optimal policy under simultaneous voting is also optimal under sequential voting with order $\{1, ..., n\}$. We next argue that optimal policy in sequential voting is order-independent. Let $\{1, ..., n\}$ and δ such that $\delta(i) \neq i$ be two voting orders. Let us rewrite IC^a-i and IC^a-$\delta(i)$ in the following form respectively:

$$\left(\sum_{\Theta_i^H} f(\theta)\prod_{j=1}^{i-1}\pi_j(\theta_j)\pi_i(H)\prod_{j=i+1}^{n}\pi_j(\theta_j)\right) \geq$$
$$\ell_i\left(\sum_{\Theta_i^L} f(\theta)\prod_{j=1}^{k-1}\pi_j(\theta_j)\pi_i(L)\prod_{j=k+1}^{n}\pi_j(\theta_j)\right). \quad (\text{IC}^a\text{-}i)$$

$$\left(\sum_{\Theta_i^H} f(\theta)\prod_{j=1}^{\delta(i)-1}\pi_j(\theta_j)\pi_i(H)\prod_{j=\delta(i)+1}^{n}\pi_j(\theta_j)\right) \geq$$
$$\ell_i\left(\sum_{\Theta_i^L} f(\theta)\prod_{j=1}^{\delta(i)-1}\pi_j(\theta_j)\pi_i(L)\prod_{j=\delta(i)+1}^{n}\pi_j(\theta_j)\right). \quad (\text{IC}^a\text{-}\delta(i))$$

The set of other voters R_{-i} is the same despite the order of play for R_i. If R_i is offered the same policy in both sequences, IC^a-i and IC^a-$\delta(i)$ become equivalent to each-other and to:

$$\left(\sum_{\Theta_i^H} f(\theta)\pi_i(H)\Pr\left(R_{-i}\text{ approve}|\theta\right)\right) - \ell_i\left(\sum_{\Theta_i^L} f(\theta)\pi_i(L)f\left(R_{-i}\text{ approve}|\theta\right)\right) \geq 0.$$

Therefore the IC^a constraint for a voter R_i is order-independent. The objective of the sender is

$$\max_{\pi_i(H),\pi_i(L)} \sum_{\Theta} f(\theta) \prod_i \pi_i(\theta_i)$$

This objective is also order-independent (the product of approval probabilities is commutative). Therefore, if $((\pi_i(H), \pi_i(L)))_i$ is a solution to the original individual persuasion problem with order $\{1, ..., n\}$, $\{(\pi_{\delta(i)}(H), \pi_{\delta(i)}(L))\}_i$ is also a solution to the new problem with order δ.

The reasoning for general persuasion is very similar. First, it is straightforward that IC$^a_{sim}$-i is equivalent to IC$^a_{seq}$-i and IC$^r_{sim}$-i to IC$^r_{seq}$-i. Therefore the optimal simultaneous policy remains optimal under sequential voting with order $\{1, ..., n\}$. Secondly, each voter, despite her rank in the sequence, only cares about $\pi(\hat{d}^a|\theta)$ and $\pi(\hat{d}^{i,r}|\theta)$. Upon observing $(i-1)$ preceding approvals, R_i is sure that the generated recommendation is in $D^{i-1,a} = \{\hat{d} : \hat{d}_j = 1 \text{ for } j = 1, ..., i-1\}$. Yet she only cares about those elements in $D^{i-1,a}$ for which $\hat{d}_j = 1$ for voters in $\{R_{i+1}, ..., R_n\}$ as well. Hence, her IC^a is order-independent: $\left(\sum_{\Theta_i^H} f(\theta)\pi(\hat{d}^a|\theta)\right) \geq \ell_i \left(\sum_{\Theta_i^L} f(\theta)\pi(\hat{d}^a|\theta)\right)$. So is IC^r-$i$ as well. The objective of the sender is also order-independent:

$$\max_{\pi(\cdot|\theta)} \sum_{\Theta} f(\theta) \prod_i \pi(\hat{d}^a|\theta).$$

For any recommendation $\hat{d}$, let $\delta(\hat{d})$ be the permuted recommendation such that $\hat{d}_i = \hat{d}_{\delta(i)}$; in particular, $\delta(\hat{d}^a) = \hat{d}^a$. Therefore, if $(\pi(\hat{d}|\theta))_{\hat{d},\theta}$ is a solution to the original general persuasion problem with order $\{1, ..., n\}$, π' such that $\pi'(\delta(\hat{d})|\theta) = \pi(\hat{d}|\theta)$ for any $\hat{d}$ and θ is a solution to the general persuasion problem under δ. The sender's payoff is the same from both π and π'. $\square$

Proof for proposition 6.3. Let $\hat{D}_i^a$ denote the set of recommendation profiles under which exactly k voters are recommended to approve and R_i is among them, and $\hat{D}_i^r$ the set of recommendation profiles under which exactly $k-1$ voters are recommended to approve and R_i is not among them. We design a full support policy as follows. For any i and any $\hat{d} \in \hat{D}_i^a$, we let

$$\Pr(\hat{d} \mid \theta) = \begin{cases} \varepsilon^2, & \text{if } \theta \neq \theta^H \\ \varepsilon, & \text{if } \theta = \theta^H. \end{cases}$$

Here, ε is a small positive number. This ensures that whenever R_i is recommended to approve, her belief of being H conditional on being pivotal (i.e., conditional on

$\hat{d} \in \hat{D}_i^a$) is sufficiently high. So IC^a-i is satisfied. For any i and any $\hat{d} \in \hat{D}_i^r$, we let

$$\Pr(\hat{d} \mid \theta) = \begin{cases} \varepsilon^2, & \text{if } \theta \neq \theta^L \\ \varepsilon, & \text{if } \theta = \theta^L. \end{cases}$$

This ensures that whenever R_i is recommended to reject, her belief of being L conditional on being pivotal is sufficiently high. So IC^r-i is satisfied as well. For any recommendation profile $\hat{d} \neq \hat{d}^a$ such that $\hat{d} \notin \hat{D}_i^a \cup \hat{D}_i^r$ for any i, we let $\Pr(\hat{d} \mid \theta) = \varepsilon^2$. For each state profile θ, once we deduct the probabilities of the recommendation profiles specified above, the remaining probability is assigned to the unanimous approval recommendation $\hat{d}^a$. This construction ensures that the policy has full support. As ε goes to zero, the probability that the project is approved approaches one. $\square$

Lemma B.1. *If f is exchangeable and affiliated, then either $f(\theta^H) + f(\theta^L) = 1$ or f has full support.*

Proof. Since f is exchangeable, $f(\theta)$ depends only on the number of high-state voters in θ. For any $k \in \{0, ..., n\}$, we let p_k denote $f(\theta)$ when exactly k voters' states are high in θ. Due to affiliation, for any $2 \leq k \leq n$ we have $p_k p_{k-2} \geq p_{k-1}^2$.

We first show that if $f\left(\theta^H\right) = 0$ (that is $p_n = 0$), then $p_k = 0$ for any $k \geq 1$. This is because p_k being zero implies that p_{k-1} being zero for any $2 \leq k \leq n$. Given the presumption that $p_n = 0$, it must be true that $p_k = 0$ for $k \geq 1$. The only possibility is that $f(\theta^L) = 1$.

We then show that if $f\left(\theta^L\right) = 0$ (that is $p_0 = 0$), then $p_k = 0$ for any $k \leq n-1$. This is because p_{k-2} being zero implies that p_{k-1} being zero for any $2 \leq k \leq n$. The only possibility is that $f(\theta^H) = 1$.

Suppose that both $f\left(\theta^H\right)$ and $f\left(\theta^L\right)$ are strictly positive. We next show that either $p_k = 0$ for all $1 \leq k \leq n-1$ or $p_k > 0$ for all $1 \leq k \leq n-1$. Suppose there exists some k' such that $p_{k'} = 0$. Then, applying the inequality that $p_k p_{k-2} \geq p_{k-1}^2$, we conclude that p_k must be zero for any $1 \leq k \leq n-1$. Therefore, either f has full support or $f\left(\theta^H\right) + f\left(\theta^L\right) = 1$. $\square$

Proof for proposition 6.4. Suppose that the sender needs k approvals. We divide the set of the state profiles Θ into three subsets. The first subset is denoted by Θ^k, which contains all the state profiles such that exactly k voters' states are H. The second subset is $\{\theta^L\}$, which contains a unique state profile such that all voters' states are L. The third subset includes the rest of the state profiles, i.e., $\Theta \setminus \left(\Theta^k \cup \{\theta^L\}\right)$. For any $\theta \in \Theta^k$, we let

$$\pi_i(\theta) = \begin{cases} 1-\varepsilon_1, & \text{if } \theta_i = H \\ \varepsilon_2, & \text{if } \theta_i = L. \end{cases}$$

For θ^L, we let $\pi_i(\theta^L) = 1-\varepsilon_3$. For any $\theta \in \Theta \setminus \left(\Theta^k \cup \{\theta^L\}\right)$, we let $\pi_i(\theta) = 1-\varepsilon_4$. We first show that IC^a-i is satisfied. For any $\theta \in \Theta^k \cap \Theta_i^H$, the probability that exactly k voters including R_i are recommended to approve is $(1-\varepsilon_1)^k(1-\varepsilon_2)^{n-k} + \mathcal{O}\left((1-\varepsilon_1)^{k-1}(1-\varepsilon_2)^{n-k-1}\varepsilon_1\varepsilon_2\right)$. For any $\theta \in \Theta^k \cap \Theta_i^L$, the probability that exactly k voters including R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_1)^{k-1}(1-\varepsilon_2)^{n-k-1}\varepsilon_1\varepsilon_2\right)$. For θ^L, the probability that exactly k voters including R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_3)^k\varepsilon_3^{n-k}\right)$. Similarly, for any $\theta \in \Theta \setminus \left(\Theta^k \cup \{\theta^L\}\right)$, the probability that exactly k voters including R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_4)^k\varepsilon_4^{n-k}\right)$. When $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ are small enough, R_i puts most of the weight on the event that $\theta \in \Theta^k \cap \Theta_i^H$ when she is recommended to approve and she conditions on being pivotal. It is obvious that R_i is willing to approve since her state is H.

We next show that IC^r-i is satisfied as well. For any $\theta \in \Theta^k \cap \Theta_i^H$, the probability that exactly $k-1$ voters excluding R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_1)^{k-1}\varepsilon_1(1-\varepsilon_2)^{n-k}\right)$. For any $\theta \in \Theta^k \cap \Theta_i^L$, the probability that exactly $k-1$ voters excluding R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_1)^{k-1}\varepsilon_1(1-\varepsilon_2)^{n-k}\right)$. For θ^L, the probability that exactly $k-1$ voters excluding R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_3)^{k-1}\varepsilon_3^{n-k+1}\right)$. Similarly, for any $\theta \in \Theta \setminus \left(\Theta^k \cup \{\theta^L\}\right)$, the probability that exactly $k-1$ voters excluding R_i are recommended to approve is $\mathcal{O}\left((1-\varepsilon_4)^{k-1}\varepsilon_4^{n-k+1}\right)$. If we let $\varepsilon_1, \varepsilon_4$ approach zero at a much faster rate than ε_3, R_i puts most of the weight on the event that $\theta = \theta^L$ when she is recommended to reject and she conditions on being pivotal. It is obvious that R_i is willing to reject since her state is L given θ^L. This completes the construction. $\square$

Proof for proposition 6.5. Suppose there exists an individual policy $(\pi_i(H), \pi_i(L))_{i=1}^n$ which is the limit of a sequence of full-support incentive-compatible policies and ensures that the project is approved for sure. We first argue that $\pi_i(H) \geq \pi_i(L)$. Pick any full-support policy $(\tilde{\pi}_i(H), \tilde{\pi}_i(L))_{i=1}^n$ along the sequence. The following IC^a and IC^r constraints must hold for each R_i:

$$\Pr(\theta_i = H|k-1 \text{ approve})\tilde{\pi}_i(H) \geq \Pr(\theta_i = L|k-1 \text{ approve})\ell_i\tilde{\pi}_i(L),$$
$$\Pr(\theta_i = H|k-1 \text{ approve})(1-\tilde{\pi}_i(H)) \leq \Pr(\theta_i = L|k-1 \text{ approve})\ell_i(1-\tilde{\pi}_i(L)),$$

we obtain that $\tilde{\pi}_i(H)/\tilde{\pi}_i(L) \geq (1-\tilde{\pi}_i(H))/(1-\tilde{\pi}_i(L))$. This implies that $\tilde{\pi}_i(H) \geq \tilde{\pi}_i(L)$ for each R_i. This must hold for all policies along the sequence, so $\pi_i(H) \geq$

$\pi_i(L)$ for each R_i.

If the project is approved for sure. It is approved for sure when $\theta = \theta^L$. Therefore, there exist k voters who approve with certainty when their states are L. Given that $\pi_i(H) \geq \pi_i(L)$ for each R_i, these voters also approve with certainty when their states are H. Thus, at least k voters approve the project for sure in both states. Let R^a be the set of voters who approve for sure: suppose there is exactly k such voters. We want to show that any voter $R_i \in R^a$ prefers to reject the project when she is recommended to approve. Conditional on being pivotal, R_i knows that all the voters not in R^a have rejected while all the voters in $R^a \setminus \{R_i\}$ have approved. R_i does not get more optimistic about her state being H from the approvals by $R^a \setminus \{R_i\}$ since these voters approve regardless of their states. R_i become more pessimistic about her state being H from the rejections by voters not in R^a. Therefore, R_i's posterior belief of being H conditional on being pivotal is lower than her prior belief. R_i strictly prefers to reject. Contradiction.

The case in which more than k voters are in R^a can be analyzed in a similar manner. Suppose that there are $k' > k$ voters in R^a. This policy is the limit of a sequence of full-support incentive-compatible policies. There exists $M \in (0,1)$ such that voters not in R^a approve with probability less than M in either state H or L, i.e., $\min\{\pi_i(H), \pi_i(L)\} = \pi_i(L) \leq M$. For any small $\varepsilon > 0$, there is a full-support policy such that voters in R^a approve with probability above $1 - \varepsilon$ in both states, i.e., $\min\{\pi_i(H), \pi_i(L)\} = \pi_i(L) \geq 1 - \varepsilon$. Pick any $R_i \in R^a$. When R_i is recommended to approve, there are two types of events in which she is pivotal: (i) $k - 1$ voters in R^a approve and the rest reject; (ii) $k'' < k - 1$ voters in R^a approve, $k - 1 - k''$ voters which are not in R^a approve, and the rest reject. As ε converges to zero, for any event of type (ii), there exists an event of type (i) which is much more likely to occur. This is because voters outside R^a are much more likely to reject than those in R^a. Therefore, the belief of R_i about her state being H is mainly driven by events of type (i). Note that in these events only voters in R^a approve. R_i does not get more optimistic about her state being H from the approvals by voters in $R^a \setminus \{R_i\}$ since these voters approve regardless of their states. R_i become more pessimistic about her state being H from the disapprovals by voters not in R^a. Therefore, R_i's posterior belief of being H conditional on being pivotal is either arbitrarily close to or smaller than her prior, so she strictly prefers to reject. Therefore R_i does not obey her approval recommendation. This shows that the project cannot be approved for sure.

We next show that each voter's payoff under individual persuasion is higher than that under general persuasion. We have shown that for each R_i, $\pi_i(H) \geq \pi_i(L)$.

We next show that there exists at least one voter such that the above inequality is strict. Suppose not. Then each voter approves with the same probability in both state H and L. Therefore, each voter's posterior belief, when she conditions on being pivotal, is the same as her prior belief. It is not incentive compatible to obey the approval recommendation since each voter's own policy is uninformative as well. Therefore, at least one voter approves strictly more frequently in state H than in state L. We index this voter by i.

We next show that R_i is pivotal with strictly positive probability. Suppose not. Then for each state profile, either (i) at least k voters out of the rest $n-1$ voters approve for sure, or (ii) at least $n-k+1$ voters out of the rest $n-1$ voters reject for sure. We first argue that, if f has full support, it is not possible that case (i) holds for some state profiles, and case (ii) holds for the others. Suppose not. Suppose that there exists a state profile such that exactly $k' \geq k$ voters out of the rest $n-1$ voters approve for sure. There also exists another state profile such that exactly $k'' \geq n-k+1$ out of the rest $n-1$ voters disapprove for sure. The intersection of these two sets of voters who approve or disapprove for sure include at least $k'+k''-(n-1)$ voters. These voters approve under one state and disapprove under the other. It is easily verified that $k'+k''-(n-1) \geq k'-k+2$. If we begin with the previous state profile under which exactly k' voters approve, we can flip the states and the decisions of $k'-k+1$ such voters so that exactly $k-1$ voters approve for sure. This makes R_i's decision pivotal, contradicting the presumption that R_i is never pivotal. Therefore, it has to be true that either case (i) holds for all state profiles or case (ii) holds for all state profiles. If f does not have full support (or equivalently the voters' states are perfectly correlated), then it is possible that case (i) holds for one state profile and case (ii) holds for the other. Given that $\pi_i(H) \geq \pi_i(L)$ for each R_i, the only possibility is that case (i) holds for θ^H and case (ii) holds for θ^L. Note that each voter obtains the highest possible payoff which is strictly positive. The statement of the proposition holds. Therefore, we can focus on the case in which either case (i) holds for both θ^H, θ^L or case (ii) holds for both θ^H, θ^L.

If case (ii) holds for all θ, this is clearly sub-optimal for the sender since he obtains a payoff of zero. Therefore, we assume that case (i) holds for all θ. However, we have argued previously that the project is not approved for sure. Contradiction. Therefore, R_i is pivotal with strictly positive probability. For R_i, a high state project is more likely to be approved than a low one. Due to affiliation, other voters benefit from the selection effect of R_i.

This shows that for any voter, a high state project is more likely to be approved than a low state one. Individual persuasion is better than general persuasion. $\square$

B.2 Optimal policy when assumption 1 fails

Our results remain intact when we add voters who prefer to approve ex ante. Once the sender designs the optimal policy for those voters who are reluctant to approve, those who prefer to approve ex ante become more optimistic about their own states. The sender simply recommends that they approve all the time. This is the case under both general and individual persuasion.

Proposition B.1. *Suppose that voter $n+1$ prefers to approve ex ante. Under both general and individual persuasion, given the optimal policy for the first n voters, voter $n+1$ is willing to rubber-stamp the first n voters' approval decision.*

Proof of proposition B.1. This result holds for individual persuasion since each voter is weakly more optimistic about her state conditional on the others' approval. Therefore, voter $n+1$ is more optimistic about her state when she conditions on the others' approvals. We thus focus on general persuasion. There are $n+1$ voters. Let $\tilde{f}$ be the distribution of these voters' states. The last voter is *convinced*, i.e., she approves given the prior belief $\tilde{f}$:

$$\ell_{n+1} \leq \frac{\sum_{\theta \in \{H,L\}^n} \tilde{f}(\theta, H)}{\sum_{\theta \in \{H,L\}^n} \tilde{f}(\theta, L)}. \tag{1}$$

Let f denote the distribution of the first n voters' states. We thus have $f(\theta) = \tilde{f}(\theta, H) + \tilde{f}(\theta, L)$. Let $(\pi(\hat{d}^a|\theta))_{\theta \in \{H,L\}^n}$ be an optimal policy with the first n voters. We want to construct a policy for $n+1$ voters $(\tilde{\pi}(\hat{d}^a|\tilde{\theta}))_{\tilde{\theta} \in \{H,L\}^{n+1}}$ such that the sender's payoff stays the same as when he faces only n voters. To ease notation, we write $\pi(\theta)$ for $\pi(\hat{d}^a|\theta)$ and $\tilde{\pi}(\tilde{\theta})$ for $\tilde{\pi}(\hat{d}^a|\tilde{\theta})$.

We let $\tilde{\pi}(\theta, H) = \tilde{\pi}(\theta, L) = \pi(\theta)$. Under this policy, the convinced voter approves whenever the first n voters approve. Then, (i) the sender's payoff stays the same; (ii) the first n voters' IC constraints are still satisfied. We only need to show that the convinced voter's IC constraint is satisfied, i.e.,

$$\ell_{n+1} \leq \frac{\sum_{\theta \in \{H,L\}^n} \tilde{f}(\theta, H)\tilde{\pi}(\theta, H)}{\sum_{\theta \in \{H,L\}^n} \tilde{f}(\theta, L)\tilde{\pi}(\theta, L)} = \frac{\sum_{\theta \in \{H,L\}^n} \tilde{f}(\theta, H)\pi(\theta)}{\sum_{\theta \in \{H,L\}^n} \tilde{f}(\theta, L)\pi(\theta)}. \tag{2}$$

Next, we want to show that the RHS of (2) is larger than that of (1). This will complete the proof.

Given the optimal policy $\pi(\cdot)$ for n voters, we let π^k be the average probability of unanimous approval for state profiles with k high states:

$$\pi^k := \frac{\sum_{|\theta|=k} f(\theta)\pi(\theta)}{\sum_{|\theta|=k} f(\theta)} = \frac{\sum_{|\theta|=k} \pi(\theta)}{\#\{\theta : |\theta| = k\}}.$$

Here $|\theta|$ is the number of high states in θ. We want to show that $\pi^k \geq \pi^{k-1}$ for $k \in \{1, ..., n\}$, that is, a profile with more high states is *on average* more likely to be approved than that with fewer high states. This, combined with the affiliation assumption, implies that the RHS of (2) is higher than that of (1). Intuitively, if $\pi^k \geq \pi^{k-1}$ for all $k \in \{1, ..., n\}$, then the convinced voter is more optimistic about her state when she conditions on the approval decision by the first n voters.

We illustrate the argument for $\pi^k \geq \pi^{k-1}$ through an example. Suppose that $n = 3$, $k = 2$. We first argue that

$$\begin{aligned} &\pi(HHL) \geq \pi(HLL), \quad \pi(HLH) \geq \pi(HLL), \quad \pi(LHH) \geq \pi(LHL), \\ &\pi(HHL) \geq \pi(LHL), \quad \pi(HLH) \geq \pi(LLH), \quad \pi(LHH) \geq \pi(LLH). \end{aligned} \tag{3}$$

Suppose not. Suppose that, for instance, $\pi(HHL) < \pi(HLL)$. We can increase $\pi(HHL)$ by $\epsilon > 0$ and decrease $\pi(HLL)$ by $\epsilon' > 0$ such that

$$f(HHL)\epsilon = f(HLL)\epsilon'.$$

This change will not affect the sender's payoff. It will not affect the incentive of any voter who has the same state in HHL and HLL, i.e., R_1's and R_3's IC constraints are still satisfied. The change will only make R_2's IC constraint easier to satisfy. Summing up inequalities in (3) and simplifying, we have shown that $\pi^k \geq \pi^{k-1}$ for $k = 2$. The argument for any n and $k \in \{1, ..., n\}$ is similar. This completes the proof. $\square$

B.3 Full-support direct obedient policies

Direct obedient policies

Let $\pi : \Theta \to \Delta\left(\prod_{i=1}^n S_i\right)$ be an arbitrary information policy. A mixed strategy for R_i in the induced voting game is given by $\sigma_i : \Pi \times S_i \to \Delta(\{0, 1\})$. Let $\sigma_i(s_i) := \sigma_i(\pi, s_i)$ denote the probability that $d_i = 1$ upon observing s_i. We claim that given any policy π and any profile of equilibrium strategies $(\sigma_i)_{i=1}^n$ we can construct a direct obedient policy that implements the same outcome as the original policy. Consider the direct policy $\tilde{\pi} : \Theta \to \Delta(\{0, 1\}^n)$ such that for any $\hat{d}$ and any $\theta \in \Theta$,

$$\tilde{\pi}(\hat{d}|\theta) = \sum_s \pi(s|\theta) \prod_{i=1}^n \left(\hat{d}_i \sigma_i(s_i) + (1 - \hat{d}_i)(1 - \sigma_i(s_i))\right).$$

Claim 1. *The direct policy π satisfies $IC^{\mathfrak{a}}$-i and $IC^{\mathfrak{r}}$-i for any i.*

Proof. Let us first show that voter R_i obeys an approval recommendation:

$$\begin{aligned}
&\sum_{\Theta_i^H} f(\theta)\tilde{\pi}(\hat{d}^a|\theta) - \ell_i \sum_{\Theta_i^L} f(\theta)\tilde{\pi}(\hat{d}^a|\theta) \\
&= \sum_{\Theta_i^H} f(\theta)\Big(\sum_s \pi(s|\theta)\prod_{i=1}^n \sigma_i(s_i)\Big) - \ell_i \sum_{\Theta_i^L} f(\theta)\Big(\sum_s \pi(s|\theta)\prod_{i=1}^n \sigma_i(s_i)\Big) \\
&= \sum_s \prod_i \sigma_i(s_i)\left\{\sum_{\Theta_i^H} f(\theta)\pi(s|\theta) - \ell_i \sum_{\Theta_i^L} f(\theta)\pi(s|\theta)\right\} \\
&= \sum_{s_i\in S_i} \sigma_i(s_i)\left(\sum_{s_{-i}}\prod_{j\neq i}\sigma_j(s_j)\left\{\sum_{\Theta_i^H} f(\theta)\pi(s_i,s_{-i}|\theta) - \ell_i \sum_{\Theta_i^L} f(\theta)\pi(s_i,s_{-i}|\theta)\right\}\right) \geq 0.
\end{aligned}$$

The last inequality follows from the fact that if $\sigma_i(s_i) \in (0,1)$ (resp., $\sigma_i(s_i) = 1$) for some s_i then the term in brackets is zero (resp., strictly positive). Hence IC^a-i is satisfied. Similar reasoning shows that IC^r constraints are satisfied as well by $\tilde{\pi}$,

$$\begin{aligned}
&\sum_{\Theta_i^H} f(\theta)\tilde{\pi}(\hat{d}^{r,i}|\theta) - \ell_i \sum_{\Theta_i^L} f(\theta)\tilde{\pi}(\hat{d}^{r,i}|\theta) \\
&= \sum_{s_i\in S_i} (1-\sigma_i(s_i))\left(\sum_{s_{-i}}\prod_{j\neq i}\sigma_j(s_j)\left(\sum_{\Theta_i^H} f(\theta)\pi(s_i,s_{-i}|\theta) - \ell_i \sum_{\Theta_i^L} f(\theta)\pi(s_i,s_{-i}|\theta)\right)\right).
\end{aligned}$$

If $\sigma_i(s_i) \in (0,1)$ (resp., $\sigma_i(s_i) = 0$) for some s_i, the term in brackets is zero (resp., strictly negative). Therefore, $\tilde{\pi}$ is obedient. □

Full-support policies

Our analysis focuses on policies that are the limit of some sequence of full-support obedient policies. For this purpose, let us define a full-support policy for each mode of persuasion. A general policy π is a full-support general policy if $\pi(\hat{d}|\theta) \in (0,1)$ for any $\hat{d} \in \{0,1\}^n$ and for any $\theta \in \Theta$. This means that for any given state profile, all recommendation profiles are sent with strictly positive probability. Similarly, a full-support independent general policy $(\pi_i(\cdot))_i$ is such that for each voter R_i, $\pi_i(\theta) \in (0,1)$ for any $\theta \in \Theta$. That is, in each state profile, each voter is recommended to approve and to reject with strictly positive probability. An individual policy $(\pi_i))_i$ has full support if for each R_i, the probability of R_i receiving each recommendation for each individual state is bounded away from zero and one, i.e. for each R_i, $\pi_i(\theta_i) \in (0,1)$ for each $\theta_i \in \{H, L\}$. That is, for each individual state θ_i, each voter is recommended to approve and to reject with strictly positive probability.

B.4 Individual persuasion with two voters

The optimal policy when the states are perfectly correlated or independent is given by proposition 4.1, so here we focus on imperfectly correlated states. Without loss, it is assumed that $\ell_1 > \ell_2$. Based on lemma 4.1 and lemma 4.2, the optimal policy features $\pi_1(H) = \pi_2(H) = 1$ and at least one voter has a binding IC^a constraint.

We first want to argue that the stricter voter's IC^a-1 constraint must bind. Suppose not. Then IC^a-2 must bind. We can solve for $\pi_2(L)$ from this binding constraint:

$$\pi_2(L) = \frac{f(LH)\pi_1(L) + f(HH)\pi_1(H)}{\ell_2(f(HL)\pi_1(H) + f(LL)\pi_1(L))}.$$

Substituting $\pi_1(H) = \pi_2(H) = 1$ and $\pi_2(L)$ into the objective of the sender, we obtain the following objective:

$$\frac{f(HH)(1+\ell_2) + f(LH)\pi_1(L)(1+\ell_2)}{\ell_2},$$

which strictly increases in $\pi_1(L)$. Therefore, the sender finds it optimal to set $\pi_1(L)$ as high as possible. This means that either IC^a-1 constraint binds or $\pi_1(L) = 1$. Given the presumption that IC^a-1 constraint does not bind, it has to be the case that $\pi_1(L) = 1$. This policy essentially provides the more lenient voter with an informative policy and asks the stricter voter to approve with probability 1. We argue that this policy cannot be incentive-compatible, because IC^a-2 binds but R_1 is asked to rubber-stamp. Given that R_1 is stricter than R_2, R_1 learns about her state indirectly from R_2's approval, and whenever R_2 is indifferent between approving and not, R_1 must strictly prefer to reject. This shows that in the optimal policy IC^a-1 must bind.

Given that IC^a-1 always binds, we can solve for $\pi_1(L)$ as a function of $\pi_2(L)$. Substituting $\pi_1(H) = \pi_2(H) = 1$ and $\pi_1(L)$ into the sender's objective, we obtain that the objective strictly increases in $\pi_2(L)$. Therefore, the sender sets $\pi_2(L)$ as high as possible. Either IC^a-2 binds or $\pi_2(L) = 1$. We summarize the discussion above in the following lemma:

Lemma B.2. *Given two voters $\ell_1 > \ell_2$ whose states are imperfectly correlated, the optimal policy is unique. The stricter voter's IC^a-1 binds. The more lenient voter's $\pi_2(L)$ is as high as IC^a-2 allows.*

B.5 Individual persuasion with homogeneous thresholds

Consider individual persuasion under the unanimous rule when n voters have the same thresholds ℓ. Without loss, we assume that $\pi_i(L) \leq \pi_{i+1}(L)$ for $1 \leq i \leq n-1$.

In any optimal policy, the voter(s) with the highest $\pi_i(L)$ must have binding IC^a constraints. The rest have slack IC^a constraints. We also assume that $\ell > 1$.

We focus on the following class of distributions f. Nature first draws a grand state which can be either G or B. If the state is G, each voter's state is H with probability $\lambda_1 \in (1/2, 1)$. If the state is B, each voter's state is L with probability $(1-\lambda_1) \in (0, 1/2)$.[1] Conditional on the grand state, the voters' states are drawn independently. We let f_k denote the probability of a state profile with k low-state voters. We thus have $f_k = p_0\lambda_1^{n-k}(1-\lambda_1)^k + (1-p_0)(1-\lambda_1)^{n-k}\lambda_1^k$.

We first argue that the support of $(\pi_i(L))_{i=1}^n$ has at most three elements. Suppose not. Then there exist i and j such that $\pi_i(L), \pi_j(L) \in (0,1)$ and both IC^a-i and IC^a-j are slack. This violates proposition 4.3.[2] This argument also shows that the support of $(\pi_i(L))_{i=1}^n$ has at most two interior-valued elements.

Therefore, any optimal policy can be characterized by three numbers (n_0, y, x) with $n_0 \geq 0$ and $0 < y \leq x \leq 1$ such that: (i) $\pi_i(L) = 0$ for $i \in \{1, ..., n_0\}$; (ii) $\pi_{n_0+1}(L) = y$; (iii) $\pi_i(L) = x$ for $i \in \{n_0+2, ..., n\}$.[3] Only the IC^a constraints of voters with $\pi_i(L) = x$ bind:

$$\ell = \frac{\sum_{k=0}^{n-n_0-2} C_{n-n_0-2}^k f_k x^k + \sum_{k=0}^{n-n_0-2} C_{n-n_0-2}^k f_{k+1} x^k y}{\sum_{k=0}^{n-n_0-2} C_{n-n_0-2}^k f_{k+1} x^{k+1} + \sum_{k=0}^{n-n_0-2} C_{n-n_0-2}^k f_{k+2} x^{k+1} y}. \tag{IC^a}$$

The sender's payoff is the sum of the numerator and the denominator of the right-hand side.

We let λ_k^* denote the value of λ_1 such that conditional on k voters' states being high, a voter is just willing to rubber-stamp. In other words, conditional on k voters' states being high, a voter's belief of being high is exactly $\ell/(1+\ell)$. Therefore, $\lambda_1 = \lambda_k^*$ solves the following equation:

$$\frac{p_0\lambda_1^k}{p_0\lambda_1^k + (1-p_0)(1-\lambda_1)^k}\lambda_1 + \frac{(1-p_0)(1-\lambda_1)^k}{p_0\lambda_1^k + (1-p_0)(1-\lambda_1)^k}(1-\lambda_1) = \frac{\ell}{1+\ell}.$$

We let p_h denote the ex ante probability of being H, that is, $p_h = p_0\lambda_1 + (1-p_0)(1-\lambda_1)$. The domain of p_h is $(1-\lambda_1, \lambda_1)$. Assumption 1 that no voter prefers to approve ex ante is equivalent to $p_h - \ell(1-p_h) < 0$. Substituting $p_0 = (p_h - (1-\lambda_1))/(2\lambda_1 - 1)$ into the equation above, we obtain the following equation that defines λ_k^*:

$$\frac{\lambda_1^{k+1}(p_h - (1-\lambda_1)) + (1-\lambda_1)^{k+1}(\lambda_1 - p_h)}{\lambda_1^k(p_h - (1-\lambda_1)) + (1-\lambda_1)^k(\lambda_1 - p_h)} = \frac{\ell}{1+\ell}.$$

Note that λ_k^* depends on p_h, ℓ, k but not on n. The left-hand side increases in p_h,

[1] If $\lambda_1 = 1$, voters' states are perfectly correlated. If $\lambda_1 = 1/2$, voters' states are independent. Both these polar cases have been addressed in proposition 4.1.

[2] It is easy to verify that the states of any three voters are strictly affiliated for this class of distributions.

[3] If $y \neq x$, only one voter has the policy y. If instead two or more voters had y, at least two voters would have interior $\pi_i(L)$ and slack IC^a constraints, contradicting proposition 4.3.

k, an λ_1.[4] The right-hand side increases in ℓ. Therefore, λ_k^* decreases in p_h and k, and it increases in ℓ.

The sequence $(\lambda_k^*)_{k=1}^{\infty}$ is a decreasing sequence which converges to $\ell/(1+\ell)$. Each voter learns about the grand state from the information regarding other voters' states. If $\lambda_1 < \ell/(1+\ell)$, even if a voter is certain that the grand state is G, this voter is not willing to rubber-stamp the project. Therefore, if $\lambda_1 \le \ell/(1+\ell)$, no matter how many other voters' states are high, a voter is unwilling to rubber-stamp. On the other hand, for any $\lambda_1 > \ell/(1+\ell)$, there exists $k \ge 1$ such that $\lambda_1 > \lambda_k^*$. We next argue that, if $\lambda_1 > \lambda_k^*$, $n_0 < k$ in any optimal policy. Because a voter is willing to rubber-stamp if $k-1$ voters have $\pi_i(L) = 0$ and another voter has $\pi_i(L) \in (0,1)$, the sender can strictly improve his payoff if at least k voters learn their states fully. For instance, if $\lambda_1 > \lambda_1^*$, the sender is able to persuade R_2 to R_n to rubber-stamp even if he only partially reveals the state to R_1. No voter will learn her state fully, so $n_0 = 0$. For $\lambda_1 > \lambda_1^*$, we are left with two possible cases: either $y < x$ or $y = x$. In the former case, R_1's IC^a is slack. The sender provides more precise information to R_1 in order to persuade the other voters more effectively. In the latter case, all voters' IC^a constraints bind. In general, if $\lambda_1 \in (\lambda_k^*, \lambda_{k-1}^*]$, we must have $n_0 < k$.

We next show that it is not possible that both y and x are interior and they are not equal to each other.

Lemma B.3. *The support of $(\pi_i(L))_{i=1}^n$ does not have two interior values.*

Proof. The first $n_0 \ge 0$ voters learn their states fully. Voter R_{n_0+1}'s policy is $\pi_{n_0+1}(L) = y$. Subsequent voters from R_{n_0+2} to R_n have the policy $\pi_i(L) = x$. Suppose that both y and x are interior and $y < x$. We want to show that the sender can strictly improve his payoff.

The sender's payoff can be written as

$$\frac{c_0(x)(1-\lambda_1)^{n_0}(\lambda_1 - p_h)(1-\lambda_1+\lambda_1 x)(1-\lambda_1+\lambda_1 y)}{2\lambda_1 - 1} + \frac{c_1(x)\lambda_1^{n_0}(p_h - 1 + \lambda_1)(\lambda_1 + (1-\lambda_1)x)(\lambda_1 + (1-\lambda_1)y)}{2\lambda_1 - 1},$$

where

$$c_0(x) = (1-\lambda_1+\lambda_1 x)^{n-n_0-2}, \quad c_1(x) = (\lambda_1 + (1-\lambda_1)x)^{n-n_0-2}.$$

The binding IC^a constraint can be written as:

$$\frac{c_0(x)}{c_1(x)} = \frac{\lambda_1^{n_0}(\lambda_1 + p_h - 1)(\lambda_1 + (1-\lambda_1)y)(\ell(\lambda_1 - 1)x + \lambda_1)}{(1-\lambda_1)^{n_0}(\lambda_1 - p_h)(\lambda_1(y-1)+1)(\ell\lambda_1 x + \lambda_1 - 1)}.$$

[4] We fix the probability p_h that a voter's state is high, so varying λ_1 only varies the correlation of the states across voters.

Given $(\ell, p_h, \lambda_1, n, n_0)$, the binding IC^a constraint implicitly defines y as a function of x. The domain of x depends on the values of $(\ell, p_h, \lambda_1, n, n_0)$. The lowest value that x can take is denoted $\underline{x}$, which is obtained when we set y to be equal to x. The highest value of x is denoted by $\overline{x}$, which is either equal to 1 or obtained by setting y to be zero. The domain of x is $[\underline{x}, \overline{x}]$.

Suppose that the pair (x, y) solves IC^a. We can replace (x, y) with $(x + \varepsilon_x, y + \varepsilon_{y,ic})$ so that the IC^a constraint still holds as an equality. Similarly, we can replace (x, y) with $(x+\varepsilon_x, y+\varepsilon_{y,obj})$ so that the sender's payoff remains constant. We define $\varepsilon'_{y,ic}(x)$ and $\varepsilon'_{y,obj}(x)$ as follows:

$$\varepsilon'_{y,ic}(x) = \lim_{\varepsilon_x \to 0} \frac{y + \varepsilon_{y,ic} - y}{x + \varepsilon_x - x}, \quad \varepsilon'_{y,obj}(x) = \lim_{\varepsilon_x \to 0} \frac{y + \varepsilon_{y,obj} - y}{x + \varepsilon_x - x}.$$

It is easy to show that both derivatives $\varepsilon'_{y,ic}(x), \varepsilon'_{y,obj}(x)$ are negative over the domain of x. It is easily verified that $\varepsilon'_{y,ic}(x)$ is negative, since a voter with the policy x must become more optimistic about her state based on policy y after we increase x by a small amount. Therefore, the binding IC^a defines y as a decreasing function of x. It is also easily verified that $\varepsilon'_{y,obj}(x)$ is negative as well: In order to keep the sender's payoff constant, we must decrease y as we increase x.

If $\varepsilon'_{y,obj}(x) < \varepsilon'_{y,ic}(x)$ over some region of x, then the decrease in y required for IC^a to hold is smaller than the decrease in y required for the sender's payoff to stay constant. In this case, the sender can improve by increasing x. Analogously, if $\varepsilon'_{y,obj}(x) > \varepsilon'_{y,ic}(x)$ over some region of x, then the sender can improve by decreasing x. We want to argue that in the domain of x, one of the following three cases occurs: (i) $\varepsilon'_{y,obj}(x) > \varepsilon'_{y,ic}(x)$ for any $x \in [\underline{x}, \overline{x}]$; (ii) $\varepsilon'_{y,obj}(x) < \varepsilon'_{y,ic}(x)$ for any $x \in [\underline{x}, \overline{x}]$; (iii) there exists x' such that $\varepsilon'_{y,obj}(x) < \varepsilon'_{y,ic}(x)$ if $x > x'$ and $\varepsilon'_{y,obj}(x) > \varepsilon'_{y,ic}(x)$ if $x < x'$. In all three cases, the sender finds it optimal to set x either as high as possible or as low as possible. Therefore, the support of any optimal policy cannot have two interior values $x, y \in (0, 1)$ with $x \neq y$.

We next show that one of three cases occurs by examining the sign of $\varepsilon'_{y,obj}(x) - \varepsilon'_{y,ic}(x)$. The term $\varepsilon'_{y,obj}(x) - \varepsilon'_{y,ic}(x)$ is positive if and only if:

$$\begin{aligned}&\left(\frac{\lambda_1 + (1 - \lambda_1)x}{1 - \lambda_1 + \lambda_1 x}\right)^{n-n_0-1} \\ >&\frac{\left(\frac{1-\lambda_1}{\lambda_1}\right)^{n_0} (\lambda_1 - p_h)\left(\ell\lambda_1 x^2(-n + n_0 + 2) - (\lambda_1 - 1)x(n - n_0 - 1) + \lambda_1\right)}{(p_h - 1 + \lambda_1)\left(\ell(1 - \lambda_1)x^2(n - n_0 - 2) + \lambda_1 x(-n + n_0 + 1) + \lambda_1 - 1\right)}.\end{aligned} \tag{4}$$

Let us first show that the second term in the denominator is negative:

$$\ell(1 - \lambda_1)x^2(n - n_0 - 2) + \lambda_1 x(-n + n_0 + 1) + \lambda_1 - 1 < 0.$$

This inequality holds when $n = n_0 + 2$, which is the lowest value that n can take.

The derivative of the left-hand side with respect to n is $-x(\lambda_1 - \ell(1-\lambda_1)x)$. We want to argue that $\lambda_1 - \ell(1-\lambda_1)x$ is weakly positive. If $\lambda_1 \geq \ell/(1+\ell)$, $\lambda_1 - \ell(1-\lambda_1)x$ is positive for any x. If $\lambda_1 < \ell/(1+\ell)$, no voter will ever rubber-stamp. The highest value of x is obtained when we set y to be zero. It is easily verified that $x < \lambda_1/(\ell(1-\lambda_1))$ in this case. Therefore, the derivative of the left-hand side with respect to n is weakly negative, so the inequality holds for any $n \geq n_0 + 2$.

The left-hand side of (4) decreases in x whereas the right-hand side increases in x. Therefore, either the above inequality holds for any x in the domain, or it does not hold for any x in the domain, or it holds only when x is below a threshold x'. This completes the proof. □

Based on lemma B.3, we are left with four possible cases:

(i) $n_0 = 0$, $y = x \in (0,1)$. In this case, all voters have the same policy. All IC^a constraints bind. This is the only symmetric policy.

(ii) $n_0 = 0$, $y \in (0,1)$, $x = 1$. R_1's information is more precise than her IC^a constraint requires. The other voters are willing to rubber-stamp given that R_1's policy is partially informative. This case is possible if and only if $\lambda_1 > \lambda_1^*$.

(iii) $n_0 > 0$, $y = x \in (0,1]$. The sender provides fully revealing policies to R_1 through R_{n_0}.

(iv) $n_0 > 0$, $y \in (0,1)$, $x = 1$. The sender provides fully revealing policies to R_1 through R_{n_0} and partial information to R_{n_0+1}.

The theme that the sender provides more precise information to some voters in order to persuade the others more effectively is reflected in the latter three cases. Moreover, when $\lambda_1 \leq \ell/(1+\ell)$, no voter ever rubber-stamps. The optimal policy must take the form of either case (i) or case (iii). If it is optimal to set n_0 to be zero, the optimal policy is the symmetric one. For $\lambda_1 > \ell/(1+\ell)$, the policy is either $(n_0, y, 1)$ with $n_0 \geq 0$ or (n_0, y, x) with $n_0 \geq 0$ and $y = x$. In the rest of this section, we analyze the parameter regions $(\lambda_1^*, 1)$, $(\ell/(1+\ell), \lambda_1^*]$, and $(1/2, \ell/(1+\ell)]$ separately.

When $\lambda_1 \in (\lambda_1^*, 1)$, only case (i) and case (ii) are possible. The threshold λ_1^* is given by:

$$\lambda_1^* = \frac{1}{2}\left(1 + \sqrt{\frac{1+\ell-4p_h}{1+\ell}}\right).$$

We are interested in characterizing how the optimal policy varies as we increase the number of voters n. If the sender chooses a policy as in case (ii), the sender's payoff is

$$\frac{(\ell+1)(\lambda_1-p_h)(\lambda_1+p_h-1)}{\ell((\lambda_1-1)\lambda_1-p_h+1)+(\lambda_1-1)\lambda_1}. \tag{5}$$

The sender chooses y so that the other voters are willing to rubber-stamp. The payoff of the sender is given by offering the policy $\pi_1(L)=y$ to R_1 since the other voters approve for sure. If $\lambda_1=\lambda_1^*$, the sender's payoff is equal to p_h, since at this correlation level the other voters are willing to rubber-stamp only if the sender fully reveals θ_1 to R_1. As λ_1 increases, the sender's payoff increases as well. When states are perfectly correlated, i.e., $\lambda_1=1$, the sender's payoff is $(1+1/\ell)p_h$, which is the same as if he were facing R_1 alone. In case (ii) the number of voters has no impact on the sender's payoff.

If the sender chooses a policy as in case (i), the IC^a constraint can be written as

$$\frac{(\lambda_1+p_h-1)(\ell(\lambda_1-1)x+\lambda_1)}{(\lambda_1-p_h)(\ell\lambda_1 x+\lambda_1-1)}=\left(\frac{\lambda_1 x+(1-\lambda_1)}{\lambda_1+(1-\lambda_1)x}\right)^{n-1}. \tag{6}$$

This equation implicitly defines x. The sender's payoff is given by

$$\frac{(\ell+1)x(\lambda_1+p_h-1)}{\ell\lambda_1 x+\lambda_1-1}(\lambda_1+x(1-\lambda_1))^{n-1}. \tag{7}$$

We will show that x, the probability that a low-state voter approves, increases in n. Moreover, the sender's payoff decreases in n. The limit of the sender's payoff as n approaches infinity is given by

$$\frac{(\ell+1)(\lambda_1+p_h-1)\left(\frac{(\ell(\lambda_1-1)+\lambda_1)(\lambda_1+p_h-1)}{(\ell\lambda_1+\lambda_1-1)(\lambda_1-p_h)}\right)^{\frac{1-\lambda_1}{2\lambda_1-1}}}{\ell\lambda_1+\lambda_1-1}. \tag{8}$$

This limiting payoff as $n\to\infty$ is lower than the payoff in case (ii). Moreover, when n equals 2, (7) is strictly higher than (5), so the sender obtains a strictly higher payoff in case (i) than in case (ii). Therefore, for each ℓ, p_h, and $\lambda_1\in(\lambda_1^*,1)$, there exists $n'\geq 3$ such that the sender is strictly better off in case (ii) than in case (i) if and only if $n\geq n'$.

Proposition B.2. *Suppose $\lambda_1\in[\lambda_1^*,1)$. For each ℓ, p_h and λ_1, there exists $n'\geq 3$ such that for $n\geq n'$, the sender is strictly better off in case (ii) than in case (i), so case (ii) policy is uniquely optimal.*

Proof. We first show that (5) is strictly higher than (8). Then we show that (7) strictly decreases in n. This completes the proof.

The ratio of (5) over (8) is given by

$$\frac{(\ell\lambda_1+\lambda_1-1)(\lambda_1-p_h)\left(\frac{(\ell(\lambda_1-1)+\lambda_1)(\lambda_1+p_h-1)}{(\ell\lambda_1+\lambda_1-1)(\lambda_1-p_h)}\right)^{\frac{\lambda_1-1}{2\lambda_1-1}}}{\ell((\lambda_1-1)\lambda_1-p_h+1)+(\lambda_1-1)\lambda_1}. \tag{9}$$

This ratio equals one when $\lambda_1 = 1$. We want to show that this ratio is strictly above one for $\lambda_1 \in [\lambda_1^*, 1)$. The derivative of this ratio with respect to p_h is negative if and only if

$$\left(\frac{(\ell(\lambda_1 - 1) + \lambda_1)(\lambda_1 + p_h - 1)}{(\ell\lambda_1 + \lambda_1 - 1)(\lambda_1 - p_h)}\right)^{\frac{\lambda_1+1}{2\lambda_1-1}} > 0.$$

Since this inequality holds, the ratio (9) decreases in p_h. Assumption 1 ensures that $p_h < \ell/(1+\ell)$. Substituting $p_h = \ell/(1+\ell)$ into (9), the ratio equals one. Therefore, for any $p_h < \ell/(1+\ell)$, the ratio (9) is above one. This completes the proof that (5) is strictly higher than (8) for $\lambda_1 \in [\lambda_1^*, 1)$.

Now we restrict attention to the policy of case (i). We first show that the solution x to (6) increases in n. The right-hand side of (6) increases in x and decreases in n. The left-hand side decreases in x. Therefore, as n increases, x must increase as well.

We next show that (7) decreases in n. It is easily verified that $((\ell + 1)x(\lambda_1 + p_h - 1))/(\ell\lambda_1 x + \lambda_1 - 1)$ decreases in x. So it also decreases in n, since x increases in n. Therefore, if we can show that $(\lambda_1 + x(n)(1 - \lambda_1))^{n-1}$ decreases in n, then (7) must decrease in n. For the rest of this proof, we use $x(n)$ instead of x to highlight the dependence of x on n. From the analysis of the previous paragraph, we know that the left-hand side of (6) decreases in n. Therefore, the total derivative of the right-hand side of (6) with respect to n is negative. This puts an upper bound on $x'(n)$:

$$x'(n) < \frac{(\lambda_1 x(n) - \lambda_1 + 1)(\lambda_1 x(n) - \lambda_1 - x(n)) \log\left(\frac{\lambda_1(x(n)-1)+1}{\lambda_1(-x(n))+\lambda_1+x(n)}\right)}{(2\lambda_1 - 1)(n - 1)}.$$

For $(\lambda_1 + x(n)(1 - \lambda_1))^{n-1}$ to be increasing in n, the derivative $x'(n)$ must be at least:

$$x'(n) > \frac{(\lambda_1 + x(n) - \lambda_1 x(n)) \log(\lambda_1(-x(n)) + \lambda_1 + x(n))}{(\lambda_1 - 1)(n - 1)}.$$

We want to show that this is impossible. The lower bound on $x'(n)$ is higher than the upper bound if and only if

$$\frac{(\lambda_1 x(n) - \lambda_1 + 1) \log\left(\frac{\lambda_1(x(n)-1)+1}{\lambda_1(-x(n))+\lambda_1+x(n)}\right)}{2\lambda_1 - 1} - \frac{\log(\lambda_1(-x(n)) + \lambda_1 + x(n))}{1 - \lambda_1} > 0.$$

The left-hand side of the above inequality decreases in $x(n)$. Moreover, the left-hand side equals zero when $x(n)$ equals one. Therefore, the above inequality always holds. This shows that the lower bound on $x'(n)$ is indeed higher than the upper bound. Therefore, $(\lambda_1 + x(n)(1 - \lambda_1))^{n-1}$ and (7) decreases in n. $\square$

We next focus on the parameter region $\lambda_1 \in (\ell/(1 + \ell), \lambda_1^*]$. The optimal policy might take the form of case (i), (iii), or (iv). We show that case (iv) dominates case (i) and case (iii) for n large enough. This implies that some voters learn their states fully for n large enough.

Proposition B.3. *Suppose $\lambda_1 \in (\ell/(1+\ell), \lambda_1^*]$. For each ℓ, p_h and λ_1, there exists $n' \geq 3$ such that for $n \geq n'$, the sender is strictly better off in case (iv) than in case (i) and (iii), so case (iv) policy is uniquely optimal.*

Proof. For any $\lambda_1 \in (\ell/(1+\ell), \lambda_1^*]$, there exists $k \geq 1$ such that $\lambda_1 \in (\lambda_{k+1}^*, \lambda_k^*]$. We first argue that the policy in case (iv) with n_0 being k leads to a higher payoff than the symmetric policy in case (i) when n is large enough.

If the sender uses the policy in case (iv) with n_0 being k, the IC^a constraint can be written as:

$$\ell = \frac{\lambda_1^{k+1}(\lambda_1 + p_h - 1)(\lambda_1 + (1-\lambda_1)y) + (1-\lambda_1)^{k+1}(\lambda_1 - p_h)(\lambda_1 y + (1-\lambda_1))}{(1-\lambda_1)\lambda_1^k(\lambda_1 + p_h - 1)(\lambda_1 + (1-\lambda_1)y) + \lambda_1(1-\lambda_1)^k(\lambda_1 - p_h)(\lambda_1 y + (1-\lambda_1))}.$$

This allows us to solve for y. Substituting this value of y into the sender's payoff, we obtain the sender's payoff as

$$\frac{(\ell+1)(2\lambda_1 - 1)((1-\lambda_1)\lambda_1)^k(\lambda_1 - p_h)(\lambda_1 + p_h - 1)}{(\lambda_1 - 1)(\ell(\lambda_1 - 1) + \lambda_1)\lambda_1^k(\lambda_1 + p_h - 1) + \lambda_1(\ell\lambda_1 + \lambda_1 - 1)(1-\lambda_1)^k(\lambda_1 - p_h)}. \tag{10}$$

The IC^a constraint holds as an equality for some y in $[0,1]$. The right-hand side of the IC^a constraint decreases in y. After substituting $y = 1$ into the IC^a constraint, we obtain the following inequality:

$$\ell \geq \frac{\lambda_1^{k+1}(\lambda_1 + p_h - 1) + (1-\lambda_1)^{k+1}(\lambda_1 - p_h)}{\lambda_1(1-\lambda_1)^k(\lambda_1 - p_h) - (\lambda_1 - 1)\lambda_1^k(\lambda_1 + p_h - 1)}.$$

This inequality imposes an upper bound on p_h:

$$p_h \leq \frac{(1-\lambda_1)(\ell(\lambda_1 - 1) + \lambda_1)\lambda_1^k + \lambda_1(\ell\lambda_1 + \lambda_1 - 1)(1-\lambda_1)^k}{(\ell(\lambda_1 - 1) + \lambda_1)\lambda_1^k + (\ell\lambda_1 + \lambda_1 - 1)(1-\lambda_1)^k}. \tag{11}$$

The payoff from the symmetric policy in case (i) approaches (8) as n goes to infinity. The ratio of (10) over (8) is given by:

$$\frac{(2\lambda_1 - 1)(\ell\lambda_1 + \lambda_1 - 1)((1-\lambda_1)\lambda_1)^k(\lambda_1 - p_h)\left(\frac{(\ell(\lambda_1-1)+\lambda_1)(\lambda_1+p_h-1)}{(\ell\lambda_1+\lambda_1-1)(\lambda_1-p_h)}\right)^{\frac{\lambda_1-1}{2\lambda_1-1}}}{(\lambda_1 - 1)(\ell(\lambda_1 - 1) + \lambda_1)\lambda_1^k(\lambda_1 + p_h - 1) + \lambda_1(\ell\lambda_1 + \lambda_1 - 1)(1-\lambda_1)^k(\lambda_1 - p_h)}.$$

This ratio decreases in p_h given the inequality (11). Moreover, if we substitute the upper bound on p_h as in (11) into the ratio above, this ratio equals:

$$\lambda_1^k\left((1-\lambda_1)^k\lambda_1^{-k}\right)^{\frac{\lambda_1}{1-2\lambda_1}+1}. \tag{12}$$

The term above strictly decreases in λ_1 if $\lambda_1 \in (1/2, 1)$. Moreover, the limit of the term above when λ_1 goes to one is equal to one. This shows that the ratio of (10) over (8) is strictly greater than one. Therefore, the symmetric policy in case (i) is dominated by the asymmetric policy in case (iv) when n is large enough.

We next argue that the policy in case (iii) with $n_0 \leq k$ leads to a lower payoff than the policy in case (iv) when n is large enough. If the sender uses the policy in case (iii), the IC^a constraint can be written as:

$$\frac{\lambda_1^{n_0}}{(1-\lambda_1)^{n_0}} \frac{(\lambda_1 + p_h - 1)(\ell(\lambda_1 - 1)x + \lambda_1)}{(\lambda_1 - p_h)(\ell\lambda_1 x + \lambda_1 - 1)} = \left(\frac{\lambda_1 x + (1-\lambda_1)}{\lambda_1 + (1-\lambda_1)x}\right)^{n-n_0-1}.$$

This equation implicitly defines x. The limit of x as n goes to infinity is 1. The limit of the sender's payoff as n approaches infinity is given by

$$\frac{(\ell+1)(\lambda_1 + p_h - 1)\lambda_1^{n_0}\left(\frac{\lambda_1^{n_0}}{(1-\lambda_1)^{n_0}} \frac{(\ell(\lambda_1-1)+\lambda_1)(\lambda_1+p_h-1)}{(\ell\lambda_1+\lambda_1-1)(\lambda_1-p_h)}\right)^{\frac{1-\lambda_1}{2\lambda_1-1}}}{\ell\lambda_1 + \lambda_1 - 1}. \tag{13}$$

The ratio of the limit payoff in case (iii) over the limit payoff in case (i) is given by the ratio of (13) over (8):

$$\left(\frac{1}{\lambda_1} - 1\right)^{\frac{(\lambda_1-1)n_0}{2\lambda_1-1}} \lambda_1^{n_0}.$$

This ratio is strictly smaller than (12) if $n_0 < k$. This ratio is equal to (12) If $n_0 = k$. Note that (12) is strictly higher than the ratio of the payoff in case (iv) over the limit payoff in case (i). This shows that the payoff in case (iv) is strictly higher than the limit payoff in case (iii). $\square$

Let us now consider the case of a low λ_1: let $\lambda_1 \in (1/2, \ell/(1+\ell)]$. For this parameter region, even if a voter is certain that the realized grand state is G, she is still not willing to approve the project if $\lambda_1 - \ell(1-\lambda_1) < 0$. Due to this, no voter is willing to rubber-stamp the project, hence there is no voter for which $\pi_i(L) = 1$. A symmetric policy assigns the same $\pi_i(L) \in (0,1)$ for any i. An asymmetric policy assigns a fully revealing policy to a subgroup of the voters and a symmetric interior policy to the remaining voters, due to lemma B.3. Hence, any asymmetric policy is indexed by n_0, the number of voters who receive fully revealing recommendations. Let $P(n_0, n)$ denote the payoff from a policy with exactly n_0 voters with fully revealing recommendations among n voters in total.

For a policy with $n_0 \geq 0$ and $x \in (0,1)$ denoting the recommendation probability $\pi_i(L)$ for the partially informed voters, the binding IC^a is:

$$\left(\frac{\lambda_1 + (1-\lambda_1)x}{1 - \lambda_1 + \lambda_1 x}\right)^{n-n_0-1} = \frac{\lambda_1^{n_0}(p_h - 1 + \lambda_1)(\lambda_1 - \ell(1-\lambda_1)x)}{(1-\lambda_1)^{n_0}(\lambda_1 - p_h)(\lambda_1 - 1 + \ell\lambda_1 x)} \tag{14}$$

The corresponding payoff to the sender is:

$$P(n_0, n) = \frac{(\ell+1)\lambda_1^{n_0}(p_h + \lambda_1 - 1)x(\lambda_1 + (1-\lambda_1)x)^{n-n_0-1}}{\lambda_1 - 1 + \ell\lambda_1 x}.$$

The approval probability x implicitly depends on n. For any fixed $n_0 \geq 0$, $x(n)$ is increasing in n. To see this, notice that the right-hand side of (14) increases in x and decreases in n, while the left-hand side decreases in x. Therefore, an increase in n makes the right-hand side smaller, while it does not directly affect the left-hand side: for equality to hold, $x(n)$ has to increase as well.

Moreover, it is straightforward to see from the IC^a constraint that $x(n)$ increases in n_0 for a fixed group size n. Naturally, if more voters are offered fully revealing recommendations, this allows the sender to recommend the partially informed voters to approve more frequently.

The following result establishes that as $n \to \infty$, the payoff of the sender decreases in n_0; hence, for an infinitely large group, it is optimal for the sender to assign the symmetric policy with $n_0 = 0$.

Proposition B.4. *Suppose that $\lambda_1 \in (1/2, \ell/(1+\ell)]$. For any $n_0 \geq 0$,*

$$\lim_{n\to\infty} \frac{P(n_0, n)}{P(n_0+1, n)} = \frac{\ell+1}{\ell} > 1,$$

that is, for sufficiently large n, the sender's payoff is decreasing in n_0.

Proof. Let $x_0(n)$ and $x_1(n)$ denote the probability of recommendation to a low-state voter corresponding to the policies with n_0 and n_0+1 fully revealing recommendations. The ratio of payoffs is:

$$\begin{aligned}&\frac{P(n_0, n)}{P(n_0+1, n)}\\ =&\frac{\lambda_1 + (1-\lambda_1)x_0(n)}{\lambda_1}\left(\frac{\lambda_1 + (1-\lambda_1)x_0(n)}{\lambda_1 + (1-\lambda_1)x_1(n)}\right)^{n-n_0-1} \frac{x_1(n)}{x_0(n)} \frac{\ell x_0(n)\lambda_1 + \lambda_1 - 1}{\ell x_1(n)\lambda_1 + \lambda_1 - 1}.\end{aligned}$$

It follows from a comparison of the two IC^a constraints that $x_1(n) > x_0(n)$ for any n. Moreover, $(x_1(n) - x_0(n)) \to 0$ as $n \to \infty$. Both $x_1(n)$ and $x_0(n)$ tend to $\lambda_1/(1-\lambda_1)\ell$. Therefore,

$$\lim_{n\to\infty} \frac{x_1(n)}{x_0(n)} = \lim_{n\to\infty} \frac{\ell x_0(n)\lambda_1 + \lambda_1 - 1}{\ell x_1(n)\lambda_1 + \lambda_1 - 1} = 1; \quad \lim_{n\to\infty} \frac{\lambda_1 + (1-\lambda_1)x_0(n)}{\lambda_1} = \frac{\ell+1}{\ell}.$$

Hence, the limit ratio of the payoffs reduces to:

$$\lim_{n\to\infty} \frac{P(n_0, n)}{P(n_0+1, n)} = \frac{\ell+1}{\ell} \lim_{n\to\infty} \left(\frac{\lambda_1 + (1-\lambda_1)x_0(n)}{\lambda_1 + (1-\lambda_1)x_1(n)}\right)^{n-n_0-1}.$$

In order to evaluate the remaining limit term, we need to approximate the rate at which $x_1(n)$ and $x_0(n)$ converge to $\lambda_1/\ell(1-\lambda_1)$ as $n \to \infty$. From the IC^a constraint when n_0 voters receive fully revealing recommendations, we have:

$$\frac{\lambda_1}{\ell(1-\lambda_1)} - x_0(n) = \frac{(2\lambda_1 - 1)(1-\lambda_1)^{n_0-2}(\lambda_1 - p_h)w_0(n)^{n-n_0-1}}{\ell\left(\lambda_1^{n_0}(\lambda_1 + p_h - 1) + \lambda_1(1-\lambda_1)^{n_0-1}(\lambda_1 - p_h)w_0(n)^{n-n_0-1}\right)},$$

where $w_0(n) = (1 - \lambda_1 + \lambda_1 x_0(n))/(\lambda_1 + (1-\lambda_1)x_0(n))$. Since $x_0(n)$ converges to $\frac{\lambda_1}{\ell(1-\lambda_1)}$ as $n \to \infty$, there exist constants $\underline{w}_0 < \overline{w}_0 \in (0,1)$ such that $w_0(n) \in (\underline{w}_0, \overline{w}_0)$

when n is sufficiently large. Similarly, from the IC^a constraint when n_0+1 voters receive fully revealing recommendations, we have:

$$\frac{\lambda_1}{\ell(1-\lambda_1)} - x_1(n) = \frac{(2\lambda_1-1)(1-\lambda_1)^{n_0-1}(\lambda_1-p_h)w_1(n)^{n-n_0-2}}{\ell\left(\lambda_1^{n_0+1}(\lambda_1+p_h-1)+\lambda_1(1-\lambda_1)^{n_0}(\lambda_1-p_h)w_1(n)^{n-n_0-2}\right)},$$

where $w_1(n) = (1-\lambda_1+\lambda_1 x_1(n))/(\lambda_1+(1-\lambda_1)x_1(n))$. Since $x_1(n)$ converges to $\frac{\lambda_1}{\ell(1-\lambda_1)}$ as $n\to\infty$, there exist constants $\underline{w}_1 < \overline{w}_1 \in (0,1)$ such that $w_1(n) \in (\underline{w}_1, \overline{w}_1)$ when n is sufficiently large.

Notice that for any $a_0, a_1 \in (0,1)$ and any $\kappa_0, \kappa_1 > 0$,

$$\lim_{n\to\infty}\left(\frac{1-\kappa_0 a_0^n}{1-\kappa_1 a_1^n}\right)^{n-1} = 1.$$

Therefore,

$$\begin{aligned}\lim_{n\to\infty}\left(\frac{\lambda_1+(1-\lambda_1)x_0(n)}{\lambda_1+(1-\lambda_1)x_1(n)}\right)^{n-n_0-1} &= \lim_{n\to\infty}\left(\frac{1-\frac{\ell(1-\lambda_1)}{(1+\ell)\lambda_1}\left(\frac{\lambda_1}{\ell(1-\lambda_1)}-x_0(n)\right)}{1-\frac{\ell(1-\lambda_1)}{(1+\ell)\lambda_1}\left(\frac{\lambda_1}{\ell(1-\lambda_1)}-x_1(n)\right)}\right)^{n-n_0-1} \\ &= 1.\end{aligned}$$

This concludes the proof. □

An immediate implication of proposition B.4 is the following corollary, which pins down the exact limiting ratio of the payoffs from the symmetric policy and any asymmetric policy with $n_0 > 0$. This ratio depends only on ℓ and n_0: the larger the threshold ℓ, the smaller the comparative benefit from a symmetric policy.

Corollary B.4. *For any asymmetric policy with $n_0 > 0$ fully informed voters,*

$$\lim_{n\to\infty}\frac{P(0,n)}{P(n_0,n)} = \left(\frac{\ell+1}{\ell}\right)^{n_0}.$$

To sum up, we analyzed optimal individual persuasion for the case of homogeneous thresholds when the group size n is sufficiently large. We have established that when λ_1 is above $\frac{\ell}{\ell+1}$, the sender finds it optimal to rely on an asymmetric policy, that assigns different recommendation probabilities $\pi_i(L)$ across voters. For instance, if $\lambda_1 > \lambda_1^*$, the optimal policy consists of a partially informed voter and all other voters rubber-stamping; if $\lambda_1 \in (\lambda_2^*, \lambda_1^*)$, the optimal policy consists of one fully informed voter, another partially informed voters, and all other voters as rubber-stampers, and so on. If on the other hand λ_1 is below $\ell/\ell+1$, it is optimal for the sender to offer the same probability of recommendation in state L to all voters when the size group is sufficiently high. No voter is fully revealed her state for such low λ_1.

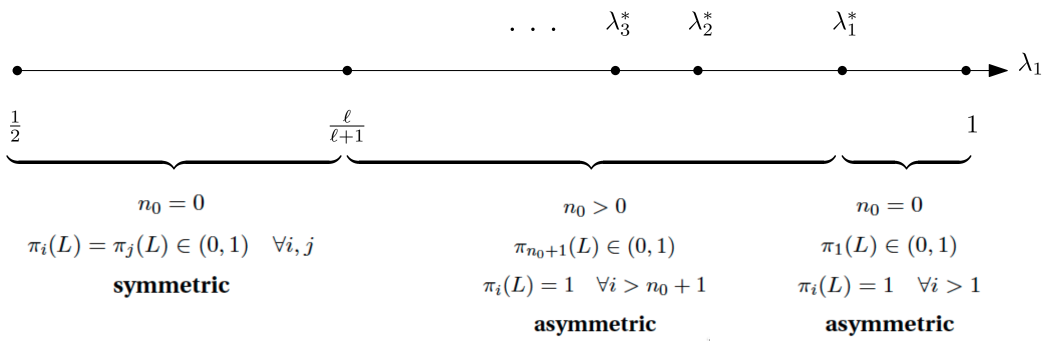

Figure 1: Optimal individual persuasion with homogeneous thresholds

B.6 Strict Ahlswede-Daykin theorem

Theorem B.5. *Suppose* $(\Gamma, \succ)$ *is a finite distributive lattice and functions* f_1*,* f_2*,* f_3*,* $f_4 : \Gamma \to \mathbb{R}^+$ *satisfy the relation that*

$$f_1(a)f_2(b) < f_3(a \wedge b)f_4(a \vee b),$$

$\forall a, b \in \Gamma$*. Furthermore, suppose that* $f_3(c) > 0$ *and* $f_4(c) > 0$ *for any* $c \in \Gamma$*. Then*

$$f_1(A)f_2(B) < f_3(A \wedge B)f_4(A \vee B),$$

$\forall A, B \subset \Gamma$*, where* $f_k(A) = \sum_{a \in A} f_k(a)$ *for all* $A \subset \Gamma, k \in \{1,2,3,4\}$*, and* $A \vee B = \{a \vee b : a \in A, b \in B\}$*,* $A \wedge B = \{a \wedge b : a \in A, b \in B\}$*.*

Proof. [5] Because Γ is a finite distributive lattice, it suffices[6] to prove that the result holds for $\Gamma = 2^N$, the lattice of subsets of the set $N = \{1, ..., n\}$ partially ordered by the inclusion relation. Then, $a \in \Gamma$ is a particular subset of N, and $A \subset \Gamma$ is a subset of subsets of N.

First, let us establish the result for $n = 1$, so $N = \{1\}$. Then $\Gamma = \{\emptyset, \{1\}\}$. Let f_k^0, f_k^1 denote the function f_k for $k = 1,2,3,4$ evaluated at $\emptyset$ and $\{1\}$ respectively. By the premise, given that $f_3^0, f_3^1, f_4^0, f_4^1 \neq 0$,

$$f_1^0 f_2^0 < f_3^0 f_4^0, \quad f_1^1 f_2^0 < f_3^0 f_4^1, \quad f_1^0 f_2^1 < f_3^0 f_4^1, \quad f_1^1 f_2^1 < f_3^1 f_4^1.$$

It is straightforward to check that the result holds for any A and B that are singletons. This leaves only the case of $A = B = \{\emptyset, \{1\}\}$. We need to show that

$$(f_1^0 + f_1^1)(f_2^0 + f_2^1) < (f_3^0 + f_3^1)(f_4^0 + f_4^1).$$

[5]This proof adapts the proof presented in Graham (1983). See: Graham, R. L. 1983. "Applications of the FKG Inequality and Its Relatives," in *Conference Proceedings, 12th Intern. Symp. Math. Programming.* Springer: 115-131.

[6]Every distributive lattice can be embedded in a powerset algebra so that all existing finite joins and meets are preserved.

If either f_1 or f_2 is zero, then the result follows trivially. So let us now consider the case in which they all nonzero. It is sufficient to consider the case for which $f_k^0 = 1$ for all k. It follows that

$$f_1^1 < f_4^1, \quad f_2^1 < f_4^1, \quad f_1^1 f_2^1 < f_3^1 f_4^1.$$

We would like to show that

$$(1 + f_1^1)(1 + f_2^1) < (1 + f_3^1)(1 + f_4^1). \tag{15}$$

If $f_4^1 = 0$, the result follows immediately. So let us consider the case for which $f_4^1 > 0$. The result we want to show becomes easier to satisfy as f_4^1: hence it is sufficient to establish it for a very low f_4^1. From the inequality $f_1^1 f_2^1 < f_3^1 f_4^1$, we know that

$$f_4^1 > \frac{f_1^1 f_2^1}{f_3^1}.$$

For some small fixed $\epsilon > 0$, let $f_4^1 = \frac{f_1^1 f_2^1}{f_3^1} + \epsilon$. From the fact that $f_1^1 < f_4^1$ and $f_2^1 < f_4^1$, it follows that

$$(f_4^1 - f_1^1)(f_4^1 - f_2^1) > 0. \tag{16}$$

We want to prove that

$$(1 + f_1^1)(1 + f_2^1) < (1 + f_3^1)\left(1 + \frac{f_1^1 f_2^1}{f_3^1} + \epsilon\right) \tag{17}$$

which is equivalent to

$${f_4^1}^2 + f_1^1 f_2^1 - f_4^1 f_1^1 - f_4^1 f_2^1 > -f_4^1 \epsilon - {f_4^1}^2 \epsilon.$$

But from 16,

$${f_4^1}^2 + f_1^1 f_2^1 - f_4^1 f_1^1 - f_4^1 f_2^1 > 0 > -f_4^1 \epsilon - {f_4^1}^2 \epsilon$$

for any $\epsilon > 0$. This establishes 17 for any arbitrarily small ϵ, hence, inequality 15 is satisfied. So the proof for $n = 1$ is concluded.

Let us now assume that the result holds for $n = m$ for some $m \geq 1$, and we would like to show that it holds for $n = m + 1$ as well. Suppose f_k, $k = 1, 2, 3, 4$ satisfy the premise of the result for $n = m + 1$ for $\Gamma = 2^{\{1,\ldots,m+1\}}$.. Let A, B be two fixed subsets of the power set $2^{\{1,\ldots,m+1\}}$. Let us define $f_k' : 2^{\{1,\ldots,m\}} \to \mathbb{R}^+$ such that

$$f_1'(a') = \sum_{a \in A, a' = a \setminus \{m+1\}} f_1(a), \quad f_2'(b') = \sum_{b \in B, b' = b \setminus \{m+1\}} f_2(b),$$

$$f_3'(w') = \sum_{w \in A \cap B, w' = w \setminus \{m+1\}} f_3(w), \quad f_4'(v') = \sum_{v \in A \cup B, v' = v \setminus \{m+1\}} f_4(v).$$

For any $a' \in 2^{\{1,\ldots,m\}}$,

$$f_1'(a') = \begin{cases} f_1(a) + f_1(a \cup \{m+1\}) & \text{if } a' \in A, a' \cup \{m+1\} \in A \\ f_1(a) & \text{if } a' \in A, a' \cup \{m+1\} \notin A \\ f_1(a \cup \{m+1\}) & \text{if } a' \notin A, a' \cup \{m+1\} \in A \\ 0 & \text{if } a' \notin A, a' \cup \{m+1\} \notin A \end{cases}$$

With such a definition,

$$f_1(A) = f_1'(2^{\{1,\ldots,m\}}).$$

Similarly, $f_2(B) = f_2'(2^{\{1,\ldots,m\}}), f_3(A \wedge B) = f_3'(2^{\{1,\ldots,m\}}), f_4(A \vee B) = f_4'(2^{\{1,\ldots,m\}})$. Now, a similar argument to the one followed for $n = 1$ with a' corresponding to $\emptyset$ before and $a' \cup \{n\}$ corresponding to $\{1\}$, gives us that

$$f_1'(a')f_2'(b') < f_3'(a' \wedge b')f_4'(a' \vee b'),$$

$\forall a', b' \in 2^{\{1,\ldots,m\}}$. But by the induction hypothesis for $n = m$,

$$f_1'(2^{\{1,\ldots,m\}})f_2'(2^{\{1,\ldots,m\}}) < f_3'(2^{\{1,\ldots,m\}})f_4'(2^{\{1,\ldots,m\}})$$

since $2^{\{1,\ldots,m\}} \wedge 2^{\{1,\ldots,m\}} = 2^{\{1,\ldots,m\}}$ and $2^{\{1,\ldots,m\}} \vee 2^{\{1,\ldots,m\}} = 2^{\{1,\ldots,m\}}$. This implies the desired result for $n = m + 1$:

$$f_1(A)f_2(B) < f_3(A \wedge B)f_4(A \vee B).$$

This concludes the proof. $\square$